# Signal Detection and Integration Theory: a Gedanken Experiment

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**Abstract**

In this letter, we propose Neyman-Pearson tests wherein we consider the use of other-than naïve integration. Specifically, we allow non-uniformity in the partitioning of the domain. This is the first study to our knowledge which considers the practical implications of a change in our knowledge of the nature of reality wherein we incorporate the fundamental status of the conscious observer. This mode of approach might allow us to set up the entire problem of communication over a feedback system in this fundamental setting.

## 1 Introduction

For background on our position on the fundamental nature of information we refer the reader to [1] and the discussion of metric perturbations and channel capacity therein. The basic position is that conscious observers exist in a sea of information. This can be arrived at if we observe that time is required to measure change, but change is required to even get a notion of time. Hence repeat-sampling of the environment, external or internal, if it yields a changed environment, is the only way to know of the passage of time.

This letter is structured as follows. In Section 2, we present selections from a paper of McShane, proposing an extension. In Section 3, we present a thought experiment utilizing the extension, and finally we conclude in Section 4.

## 2 Extending McShane's Integration Theory

In [2], McShane presented a unified theory of integration. It is the task of this section to first succinctly present the same. We quote McShane:

> DEFINITION 2.1. A real-valued function $f$ is said to be integrable over a rightclosed interval $(a, b]$ if it is defined on the closure $[a, b]$, and there is a number $J$ such that: to each positive $\in$ there corresponds a gauge $\delta$ such that for every $\delta$-fine partition $P =$

> $\{(\overline{x_1}, A_1), ..., (\overline{x_k}, A_k)\}$ of $(a, b]$ it is true that $\sum_{i=1}^{k} |f(\overline{x_i})mA_i - J| < \epsilon$. In this case, we define $\int_a^b f(x)dx = J$.

Here $mA_i$ refers to the length of $A_i$ and the $A_i$'s can be distinct. What if we relaxed the word 'every' in this definition such that for some partitions, specifically those with non-identical $A_i$'s, we obtained $J'$ which was slightly different from $J$. As per Definition 2.1, the integral would not exist. McShane mentions functions with discontinuity and the property of unboundedness as not having Darboux or Riemann integrals, but having integrals as per Definition 2.1 above. We want to generalize McShane further such that in cases where the McShane integral does not exist, the new integral does. We propose:

> DEFINITION 2.2. A real-valued function $f$ is said to be integrable over a rightclosed interval $(a, b]$ if it is defined on the closure $[a, b]$, and there is a number $J$ such that: to each positive $\in$ there corresponds a gauge $\delta$ such that for every $\delta$-fine partition $P = \{(\overline{x_1}, A_1), ..., (\overline{x_k}, A_k)\}$ of $(a, b]$ it is true that $\sum_{i=1}^{k} |f(\overline{x_i})mA_i - J| < \epsilon \pm \kappa_\epsilon$. In this case, we define $\int_a^b f(x)dx = J$.

Clearly, when $\kappa_\epsilon = 0$ we recover Definition 2.1. By providing an $\in$-dependent interval around $\in$, for every $\in$, we allow a certain fuzziness which permits the case we are interested in - that of non-uniform partitions having slightly different $J$'s.

# 3 Thought Experiment

Consider the use of Neyman-Pearson testing for antipodal signal detection in white Gaussian noise. This is a central element of feedback communication over a noisy Gaussian feedback system [3]. Generally, one would form the following integral:

$$\int_0^T Y(t)r(t)dt \tag{1}$$

and would compare it with a real number $z$ set beforehand. If the integral is greater than or less than the set real number, one would decode to the positive or the negative signal respectively. Here $T$ is the duration of the signals, $Y(t)$ is the received signal and $\{r(t), -r(t)\}$ is the set of antipodal transmission signals.

However, in light of the introduction, we can ask the question, how does one know that the time interval $[0, T]$ has elapsed? As a pure perceiver, or subject, one samples the 'world' through one's senses, or instruments-in-aid of the senses. One samples now, and finds one setting of the universe and samples again to find another setting or state. In this view, the naïve Newtonian assumption of an absolute arena of space and time is given up for a more careful consideration.

The integral of Equation (1) can be broken down as usual into the Riemann sum. This sum will contain terms involving $\triangle t$. This is where the absolute time assumption of Newton comes in. In this letter we propose that instead of uniform sampling of so-called time, we allow variable $\triangle t$'s. To concretize this, consider a box of gas molecules in random motion which can also be heated or cooled from outside randomly. Let $R$ represent the position of every molecule at the present moment and $R'$ when the subject (us) samples the box 'again'. Based on the change in the configuration of the gas molecules, observed between $R$ and $R'$, we compute how much time has elapsed. This then goes into the Riemann sum as the first $\triangle t$. The next $\triangle t$ and so on are obtained uniquely in a similar fashion using the box of gas molecules - there is a possibility that every one of them will be distinct. As a result, one obtains an integral of the type specified in Definition 2.2 of Section 2 instead of Equation (1).

## 4 Discussion

The implication of the new integral and its use in antipodal signal detection is that we can set up our detection theory to be in consonance with the fact of perception, the presence of the subject or observer. Even if the two integrals might be a match in several situations, it is more correct to define signal detection in the Definition 2.2 sense discussed herein. This can be extended to every part of Gaussian feedback communication and other communications settings. It must be kept in mind, however, that one of the limitations of this letter is that we didn't consider the mathematical viability (from an analysis standpoint [4]) and all the ramifications of our Definition 2.2. In particular, the decoding might also acquire fuzziness from the definition's fuzziness. This issue is left to be discussed in future work.

When a standard is 'agreed' upon (of which an absolute time standard is an extreme case), it demotes the primacy of the subject. This has consequences as discussed herein. In future work, we would like to quantify the difference the substitution of naïve integrals with extended McShane integrals makes in common detection settings. Protocols may be devised wherein every party operates on true subjective time, even perhaps independent of the box proposed here, but each can comprehend the other. The modern day study of local oscillators in time standards is an example of these tensions being examined in the recent technological literature [5]. Our view of time and its implications for signal detection will also have relevance for those interested in the quantum foam of spacetime [6].

## References

[1] Aman Chawla, Salvatore D. Morgera, and Arthur D. Snider. Fields, geometry, and their impact on axon interaction. *Journal of Applied Mathematics and Physics*, 9(4):751–778, 2021.

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[5] G John Dick. Local oscillator induced instabilities in trapped ion frequency standards. In *Proceedings of the 19th Annual Precise Time and Time Interval Systems and Applications Meeting*, pages 133–147.

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